Probability Summary

1. Probability scale

Probabilities are always a fraction between 0 and 1.

Zero probability means the event won't happen. Probability 1 means that the event is certain to happen.

Probabilities can be fractions like $\frac{5}{6}$, decimal fractions like 0.75 or percentages.

"One in two chance" isn't a probability, nor is 1:6.

2. Trials, events and outcomes

Tossing a coin could be a <u>trial</u>. An <u>event</u> could be 'coin lands heads side up', and you can estimate the probability of that event as $\frac{1}{2}$. An <u>outcome</u> is what you observe when the coin lands. Events and outcomes have to be SMART. 'It rains tomorrow' is not a SMART event or outcome! How could you make a SMART event about tomorrow's weather?

3. Probability formula

$$P(event)=\frac{\text{Number of ways event can happen}}{\text{Number of possible outcomes}} .$$

An example would be rolling a dice and getting a prime number (2, 3, 5)

$$P(prime)=\frac{\text{3 prime numbers between 1 and 6}}{\text{the 6 possible outcomes}}=\frac{3}{6}=\frac{1}{2}$$

4. Probability that an event does not happen

An event either happens or it does not. So the probability of the event happening and the probability of it not happening have to add to 1. So...

$$P(\text{event does not happen})=1-P(event)$$

The probability of rolling a square number on a dice is $\frac{1}{3}$

The probability of not getting a square number when you roll a dice is $1-\frac{1}{3}=\frac{2}{3}$

5. Mutually exclusive events (OR means ADD)

Mutually exclusive events cannot happen at the same time. If you pick a ball out of a bag, the ball can't be green and blue, it has to be one colour <u>or</u> the other.

You can add the probabilities of mutually exclusive events.

A bag contains 4 red balls, 5 blue balls and 3 green balls. You pick one ball...

$$P(\text{red OR green})=\frac{4+3}{12}=\frac{7}{12}$$

When you put OR between events, you can ADD the probabilities if the events are mutually exclusive.

6. Expected frequency, expectation

The word frequency gets used in different ways in probability.

The expected frequency of heads when you toss a coin 100 times is 50.

If you roll a dice 600 times, the expectation of getting a 5 is 100 times.

You multiply the number of trials by the probability of the event.

$$\text{Expectation of outcome} = \text{Number of trials} \times \text{Probability of event}$$

The photocopier has a 0.05 probability of not working. You use the copier 200 times. How many times would you expect the copier not to be working?

$$0.05 \times 200 = 10$$

You might find the copier not working 8 or 6 or 12 times, but rarely 30 or 50 times.

If someone claimed to have tossed a coin 300 times and seen *exactly* 150 heads, you would be right to be very suspicious.

Google Binomial Distribution if you have an interest in this (way beyond syllabus).

7. Relative frequency, experimental probabilities

Relative frequency is another phrase for probability when the probability is calculated from frequencies you have *observed*.

If you roll a dice 24 times and get the score 6 on five rolls, then the relative frequency of rolling a 6 is $5 \div 24 = 0.208333 \approx 0.21$

If you roll the dice a lot of times, you would expect the relative frequency of score 6 to get closer to $\frac{1}{6} \approx 0.1666\ldots$

8. Independent events (AND means MULTIPLY)

If you roll a dice and toss a coin, the dice score does not depend on which way the coin fell.

The events 'head on coin' AND 'six on dice' are independent, and you can multiply their probabilities.

$$P(\text{Head AND six}) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$

You can draw 'sample space diagrams' showing the possible outcomes...

		Dice score					
		1	2	3	4	5	6
Coin	Head	H1	H2	H3	H4	H5	H6
	Tail	T1	T2	T3	T4	T5	T6

Suppose in a game, your dice score was doubled if the coin came down heads.

Draw a new possibility space diagram showing the actual scores.

What is the probability of you scoring more than 5?