

①

Worksheet 20: solutionsBasic convention of algebra

- 1) $b^2 + b^2 + b^2 = 3b^2$ (NOT b^6 as not x)
- 2) $f + f + f + f - f = 4f - f = 3f$.
- 3) $11x - 4x = 7x$
- 4) $3b$ means $3 \times b$ so $3 \times 6 = 18$
- 5) $3a \times 5b = 15ab$ [$a \times b = ab$ in algebra]
- 6) $-7p \times 5q = -35pq$
- 7) $-5x \times -6y = 30xy$ [$- \times - = +$]
- 8) $4xy + yx = 5xy$ [$x \times y$ same as $y \times x$]
- 9) $8x^2$ [you are counting how many x^2 you have]
- 10) $4x \times 5x = 20x^2$ [$x \times x = x^2$]
- 11) $x^5 \times x^4 = x^9$ [multiply powers of same number] ^{by adding powers}
- 12) $a \times b \times c = abc$
- 13) $4xy \times 3x = 12x^2y$ [Sign, ~~letter~~ numbers, letters]
- 14) $-3xy \times 5yx = -15x^2y^2$
- 15) $20y + x$
- 16) $-15p + 20q$ or $20q - 15p$
- 17) The median is x
with ~~algebra~~ always be in same order
because x^2 gets rid of sign of x

Bidmas and neg numbers

- 1) $-3 - 5 = -8$ (same signs add give sign)
- 2) $-6 \times 7 = -42$
- 3) $15 \div -3 = -5$
- 4) $-8 + 17$ same as $17 - 8 = 9$
- 5) $-4 \times -6 = 24$ [$- \times - = +$]
- 6) $5 \times 7 + 3 \times 4 = 35 + 12 = 47$
- 7) $6 \times 3 - 5 \times 3 = 18 - 15 = 3$
- 8) $(3 \times 4) - (5 \times -2) = 12 - (-10) = 22$
- 9) $\frac{(-2)^2 - (3 \times 6)}{\sqrt{49}} = \frac{4 - 18}{7} = \frac{-14}{7} = -2$

(2)

Worksheet 20 solutions

Collect like terms

- 1) simplify $+3y + 4y + 2y + y = 10y$ ↙ count as 1y
- 2) $+4x + 3 + 2x + 8 = 6x + 11$
- 3) $+5x + 2y - 2x + 3y = 5x - 2x + 2y + 3y = 3x + 5y$
- 4) $-3q + 2p - q + 5p = 2p + 5p - 3q - q = 7p - 4q$
- 5) x^2 different to x so add separately
 $+3x^2 - 4x + x^2 - x = 4x^2 - 5x$
- 6) xy same as yx so just count them up
 $4xy + 3xy - xy = 7xy - 1xy = 6xy$
- 7) $x^2 + x - 12$ only the x terms can be added
- 8) $+x + y - x - y + 6 = 6$ (letters cancel out)
- 9) $+4x - 2y + 3x - 6y - x + 5y = 6x - 3y$
- 10) $\frac{12x - 18y}{6} = 2x - 3y$ so $a = 2$ and $b = -3$

[you divide each term separately
 so $\frac{12x - 18y}{6} = \frac{12x}{6} - \frac{18y}{6}$]

multiply out brackets

- 1) $3(4x + 5) = 12x + 15$
- 2) $2(x - 3) = 2x - 6$
- 3) $-3(2x + 1) = -6x - 3$
- 4) $-5(3x - 2) = -15x + 10$ ($-x- = +$)
- 5) $x(x + 1) = x^2 + x$
- 6) $2(2x + 1) + 3(5x + 2) = 4x + 2 + 15x + 6$ collect terms
 $= 19x + 8$
- 7) $3x(2x - 5) = 6x^2 - 15x$ ($x \times x = x^2$)
- 8) $3(x + 1) - 2(4x - 5) = 3x + 3 - 8x + 10$
 $= -5x + 13$ or $13 - 5x$
- 9) $3xy(5x + 2y) = 15x^2y + 6xy^2$
- 10) $\frac{12x - 9}{3} - \frac{8x - 20}{4} = \frac{12x}{3} - \frac{9}{3} - \frac{8x}{4} + \frac{20}{4}$
 $= 4x - 3 - 2x + 5$
 $= 2x - 8$
- $a = 2, b = -8$

Worksheet 20 solutions (3)

Factorise linear expressions

$$1) \quad 12x + 6 = 6(2x + 1)$$

$$2) \quad 4p - 12 = 4(p - 3)$$

$$3) \quad 5x + 10 = 5(x + 2)$$

$$4) \quad 18 - 6a = 6(3 - a)$$

$$5) \quad x(x - 1) \quad [\underline{x}x - \underline{x} \text{ so take an } x \text{ outside}]$$

$$6) \quad 3x^2 - 12x = 3\underline{x}x - 12\underline{x} = \underline{3x}(x - 4)$$

$$7) \quad 4xy + 12x^2 = 4\underline{xy} + 12\underline{x}x = 4x(y + 3x)$$

$$8) \quad xy^2 - x^2y = \underline{xy}y - \underline{xx}y = xy(y - x)$$

$$9) \quad 25 - 10x = 5(5 - 2x)$$

$$10) \quad \text{Top heavy} \dots \frac{3}{4}x + \frac{9}{4} = \frac{3}{4}(x + 3)$$

Multiply pairs of brackets F.O.I.L.

$$1) \quad (x+1)(x+2) = x^2 + 2x + x + 2 = \underline{x^2 + 3x + 2}$$

$$2) \quad (x+1)(x-2) = \overset{F}{x^2} + \overset{O}{-2x} + \overset{I}{x} + \overset{L}{-2} = \underline{x^2 - x - 2}$$

$$3) \quad (2x+1)(x+5) = \overset{F}{2x^2} + \overset{O}{10x} + \overset{I}{x} + \overset{L}{5} \\ = 2x^2 + 11x + 5$$

$$4) \quad (x+3)(x-3) = \overset{F}{x^2} + \overset{O}{-3x} + \overset{I}{3x} + \overset{L}{-9} = x^2 - 9$$

$$5) \quad (2x+5)(2x-5) = 4x^2 - 10x + 10x - 25 \\ = 4x^2 - 25$$

$$6) \quad (12x-2)(x+3) = 12x^2 + 36 - 2x - 6 \\ = 9x + 36 - 2x^2$$

$$7) \quad (3x-6)(4x+5) = 12x^2 + 15x - 24x - 30 \\ = 12x^2 - 9x - 30$$

$$8) \quad (4x-3)(4x+3) = 16x^2 + 12x - 12x - 9 = 16x^2 - 9$$

$$9) \quad (x+3)(x+2) = x^2 + 2x + 3x + 6 = \underline{x^2 + 5x + 6}$$

$$10) \quad (x+3)(x+1) = x^2 + x + 3x + 3 = x^2 + 4x + 3$$

$$a=1 \quad b=4 \quad c=3$$

$$11) \quad px + qx = 3x \quad \text{and} \quad pq = c \quad \text{so} \quad \text{go for } p=1, q=2 \\ (x+2)(x+1) = x^2 + 3x + 2 \quad \checkmark$$

Worksheet 20: solutions (4)

$$12) (px+q)(px-q) = p^2x^2 - \cancel{qpx} + \cancel{qpx} - q^2 = (px)^2 - q^2$$

$$p^2x^2 = (px) \times (px)$$

Factorise quadratic expression

both +

same signs

$$1) x^2 + 2x + 1 = (x + 1)(x + 1)$$

$$2) x^2 + 4x + 4 = (x + 2)(x + 2)$$

both + same sign each bracket

③ Looking for factors of +4 that add to 4 so must be 2, 2

$$3) x^2 + 2x - 3 = (x + 3)(x - 1)$$

② Largest is +

① one + one -

② Looking for two factors of -3 that add to +2 +3, 1

$$4) x^2 - 2x + 3 = (x - 3)(x + 1)$$

$$5) x^2 - 9 = (x + 3)(x - 3)$$

$$6) 6x^2 + 19x + 15$$

all + signs

$$(6x + 5)(x + 3) =$$

5x + 18x too big

$$(3x + 3)(2x + 5) =$$

6x + 15x too big

$$(2x + 3)(3x + 5) =$$

$$\text{so } 6x^2 + 19x + 15 = \underline{(2x + 3)(3x + 5)} \quad 9x + 10x = 19x \checkmark$$

[trial and error with two sets of factors]

Worksheet 20: Solutions

(5)

Use function machine

Question (1)

In	0	1	2	3	$4\frac{1}{2}$	9	-1	-7
out	5	8	11	14	$18\frac{1}{2}$	32	2	-16

$$4\frac{1}{2} \times 3 = 13\frac{1}{2} + 5 = 18\frac{1}{2}$$

Reverse boxes and operation
 $32 - 5 = 27 \div 3 = 9$

In	$1\frac{1}{2}$	x
out	$9\frac{1}{2}$	$3x + 5$

$$9\frac{1}{2} - 5 = 4\frac{1}{2} \div 3 = 1\frac{1}{2}$$

Question (2)

a) $5 \times 4 = 20 - 6 = \underline{\underline{14}}$ output

b) $-10 + 6 = -4 \div 4 = -1$ input

c) $4x - 6 = 2x$

so $2x - 6 = 0$

so $2x = 6$

$x = 3$

check $3 \times 4 = 12 - 6 = 6$.

You could try various inputs and narrow down to $x = 3$.

Worksheet 20 solutions:

(6)

Substituting into expression

1) $3 \times 7 = \underline{21}$

2) $3 \times 2 + 2 \times 5 = 6 + 10 = 16$ so $A = 16$

3) $y = 4x + 3$ $y = 4 \times 3 + 3 = 12 + 3 = \underline{15}$

4) $B = 3 \times 3 - 2 \times 8 = 9 - 16 = \underline{-7}$

5) $C = \underset{15}{(3 \times 5)} - \underset{-8}{(4 \times -2)} = 15 - (-8) = 23$

6) $2y + 3x = 2 \times 0 + 3 \times 4 = 12$

7) a) $12 = 4x + 3 \times 0$ so $x = 3$

b) $12 = 4 \times 0 + 3y$ so $y = 4$

Making expressions from words

1) $15n$

2) $20x$

3) $25x + 20y$

4) $2n - 5$

5) James is x years old

Indrajit is twice as old as James $2x$

Aaron is three years older than James $x + 3$

Keyword total means to add
symbol

$x + 2x + x + 3 = \underline{4x + 3}$

Symbol

x

$2x$

$x + 3$

Worksheet 20: solutions

(7)

Solving (mostly) linear equations

1) $3x = 15$ so divide by 3 $15 \div 3 = 5$

$x = 5$

2) $\frac{x}{10} = 2$ so $x = 10 \times 2 = \underline{20}$

3) $x + 9 = 12$ so $x = 12 - 9 = \underline{3}$

4) $x - 6 = 20$ so $x = 20 + 6 = \underline{26}$

5) $3x + 4 = 19$ so $3x = 19 - 4 = 15$

$x = 5$ ($\div 3$)

6) $5x - 6 = 4$ so $5x = 10$ and $x = 2$

7) $4x + 6 = 2$ so $4x = 2 - 6 = -4$

$x = -4/4 = -1$

Don't reverse subtract!

8) $5x + 3 = 10$ so $5x = 7$

so $x = 7/5 = 1\frac{2}{5}$ or 1.4

9) $1.2x + 0.4 = 2$ so $1.2x = 2.4$

$x = 2.4 \div 1.2 = 2$

10) $3x + 5 = 4x - 3$

subtract $3x$ both sides

$5 = x - 3$

add 3 on

$8 = x$

11) $2x + 6 = 4x - 14$

subtract $2x$ both sides

$6 = 2x - 14$

add 14

$20 = 2x$

divide by 2

$10 = x$

12) $x^2 = 16$ so $x = +4$ or $x = -4$

Worksheet 20: solutions

(8)

Change subject of a formula

1) $y = 3x$ so undo $\times 3$
 $\frac{y}{3} = x$ and formula is $x = y/3$

2) $A = C + 5$ so undo $+5$
 $A - 5 = C$ and formula is $C = A - 5$

3) $A = 4m + 5$ so undo $+5$
 $A - 5 = 4m$ and now undo $\times 4$
 $\frac{A - 5}{4} = m$ so formula is $m = \frac{A - 5}{4}$
or $\frac{1}{4}(A - 5)$

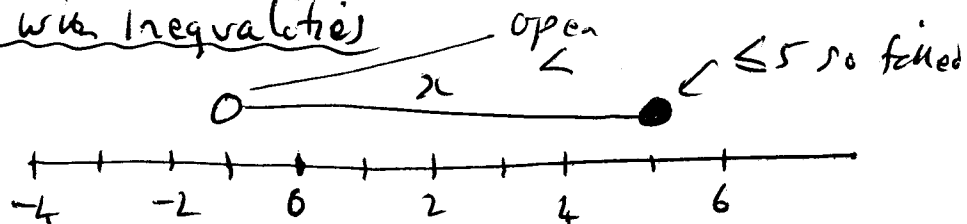
4) $A = L \times W$ undo $L \times$
 $A/L = W$ and formula is $W = A/L$

5) $y + 3x = 12$ just subtract $3x$
 $y = 12 - 3x$

6) $y = 2x + 1$ undo $+1$
 $y - 1 = 2x$ undo $2 \times$
 $\frac{y - 1}{2} = x$ so formula is $x = \frac{y - 1}{2}$
or $\frac{1}{2}(y - 1)$

Work with Inequalities

Q1

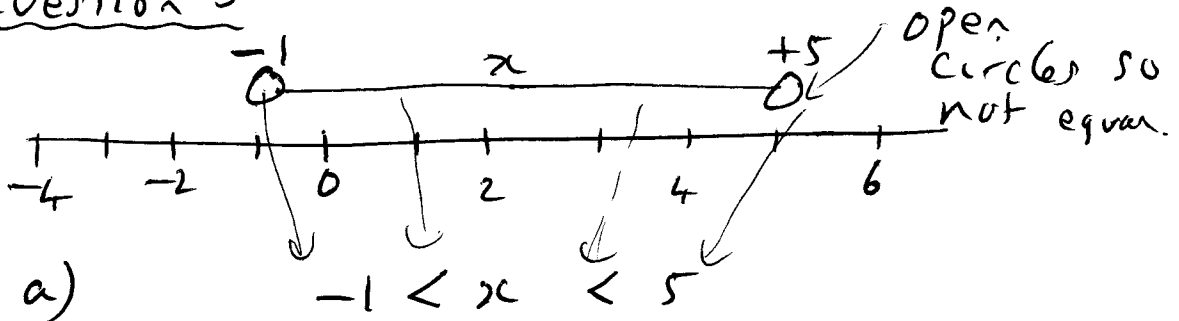


Q2) $-2 \leq y < 3$
 y can be -2 y can't be 3
 $-2, -1, 0, 1, 2$



9

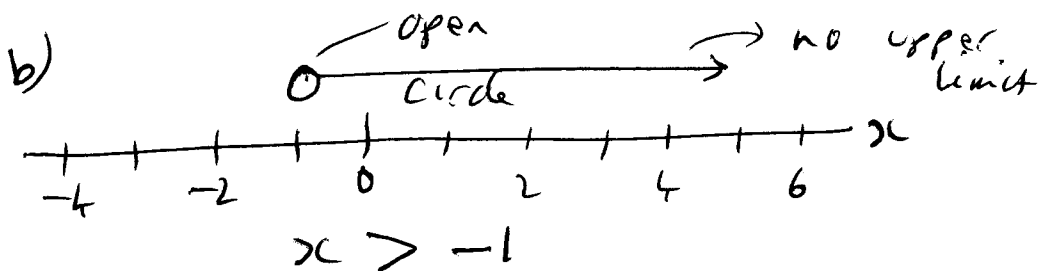
Worksheet 20: solutions
working with inequalities
Question 3



b) 0, 1, 2, 3, 4

Question 4

a) $3x + 1 < 16$ (undo +1)
 $3x < 15$ (undo x3)
 $x < 5$



c) bigger than -1 and less than 5 so
0, 1, 2, 3, 4

Solve simultaneous equation

D) Column method

Turnips	Potatoes	Cost
2	3	210
1	3	150

Difference tells us cost of a turnip

1 = 60p Turnip

So $60p + 3 \times \text{spud} = 150$

$150 - 60 = 90 \div 3 = 30p$

Potato = 30p

Worksheet 20: solutions (10)

Simultaneous equations continued

2) $3x + y = 18$ — (1) Same coefficient so eliminate y
 $x + y = 8$ (2) Same sign so we can subtract

$$\begin{array}{r} 2x = 10 \\ x = 5 \end{array}$$

substitute into (2) $5 + y = 8$ so $y = 3$.
 check in (1)

3) $4x + 2y = 14$ — (1) (add to eliminate y)
 $3x - 2y = 7$ — (2)

$$\begin{array}{r} 7x = 21 \\ x = 3 \end{array}$$

substitute into (1) to find y $4 \times 3 + 2y = 14$ $y = 1$
 check in (2) $3 \times 3 - 2 \times 1 = 7$ ✓

4) $5x + 3y = 26$ $\times 2$ (1)
 $2x + 4y = 16$ $\times 5$ multiply to find LCM.
 (eliminate x for a change)
 $10x + 6y = 52$ — x terms same sign so subtract
 $10x + 20y = 80$

$$\begin{array}{r} 14y = 28 \\ y = 2 \end{array}$$

substitute into (1)
 $5x + 3 \times 2 = 26$
 $x = 4$.

check in (2) $2 \times 4 + 4 \times 2 = 16$ ✓

5) $2x + 3y = 15$ $\times 2$
 $5x - 2y = -\frac{1}{2}$ $\times 3$

$$\begin{array}{r} 4x + 6y = 30 \\ 15x - 6y = -\frac{1}{2} \\ \hline 19x = 28\frac{1}{2} \\ x = 28\frac{1}{2} \div 19 \\ x = 1.5 \end{array}$$

~~6)~~ $4 \times 1\frac{1}{2} + 6y = 30$
 $6y = 24$
 $y = 4$

Worksheet 20 solutions

(11)

Find n^{th} term for linear sequence

1)

Term number	0	1	2	3	4
Term value	4	7	10	13	16
		+3	+3	+3	+3

So n^{th} term is $3n+4$

2) $307 - 3 = 304$ / Subtract 1st term
 what is left should be a multiple of
 the term to term difference which is

$$4 \overline{) 304}$$

Yes 304 is a multiple of 4 so
 307 is in the linear sequence

3) 1, 1, 2, 3, 5, 8, 13

Differences are not same so not
 a linear sequence

4) 2, 8, 14 Common difference is 6

Term number (n)	0	1	2	3	4
Term value	-4	2	8	14	20
		+6	+6	+6	+6

So n^{th} term is $6n-4$

5) Term number (n)	0	1	2	3	4
Term value	43	40	37	34	31
		-3	-3	-3	-3

Last term is $n=14$

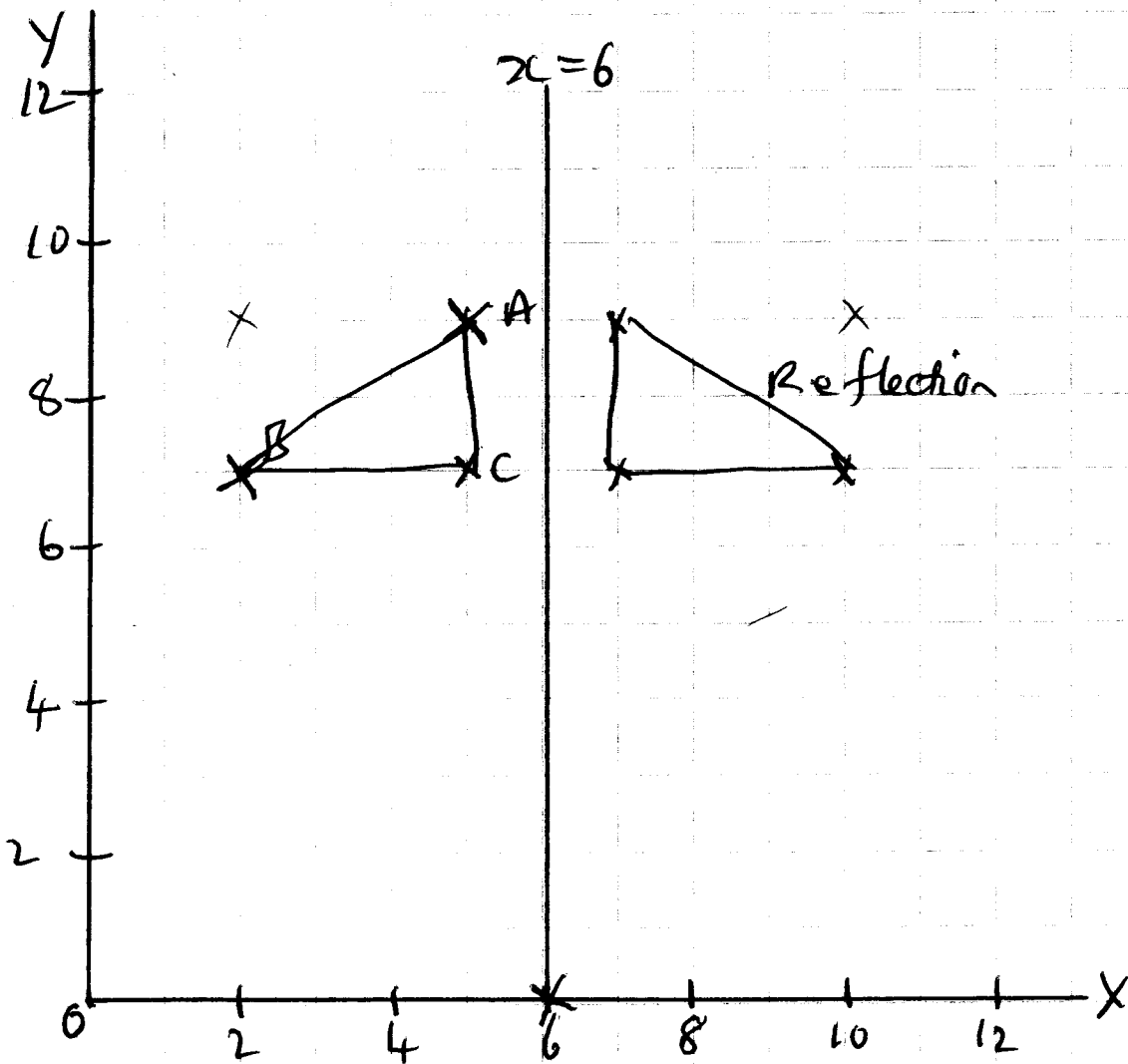
$$-3n + 43 \text{ is } n^{\text{th}} \text{ term}$$

$$-3n + 43 = 0 \Rightarrow$$

$$43 = 3n \quad n = 43/3 = 14\frac{1}{3}$$

Worksheet 20: solution
plot coordinates

(12)



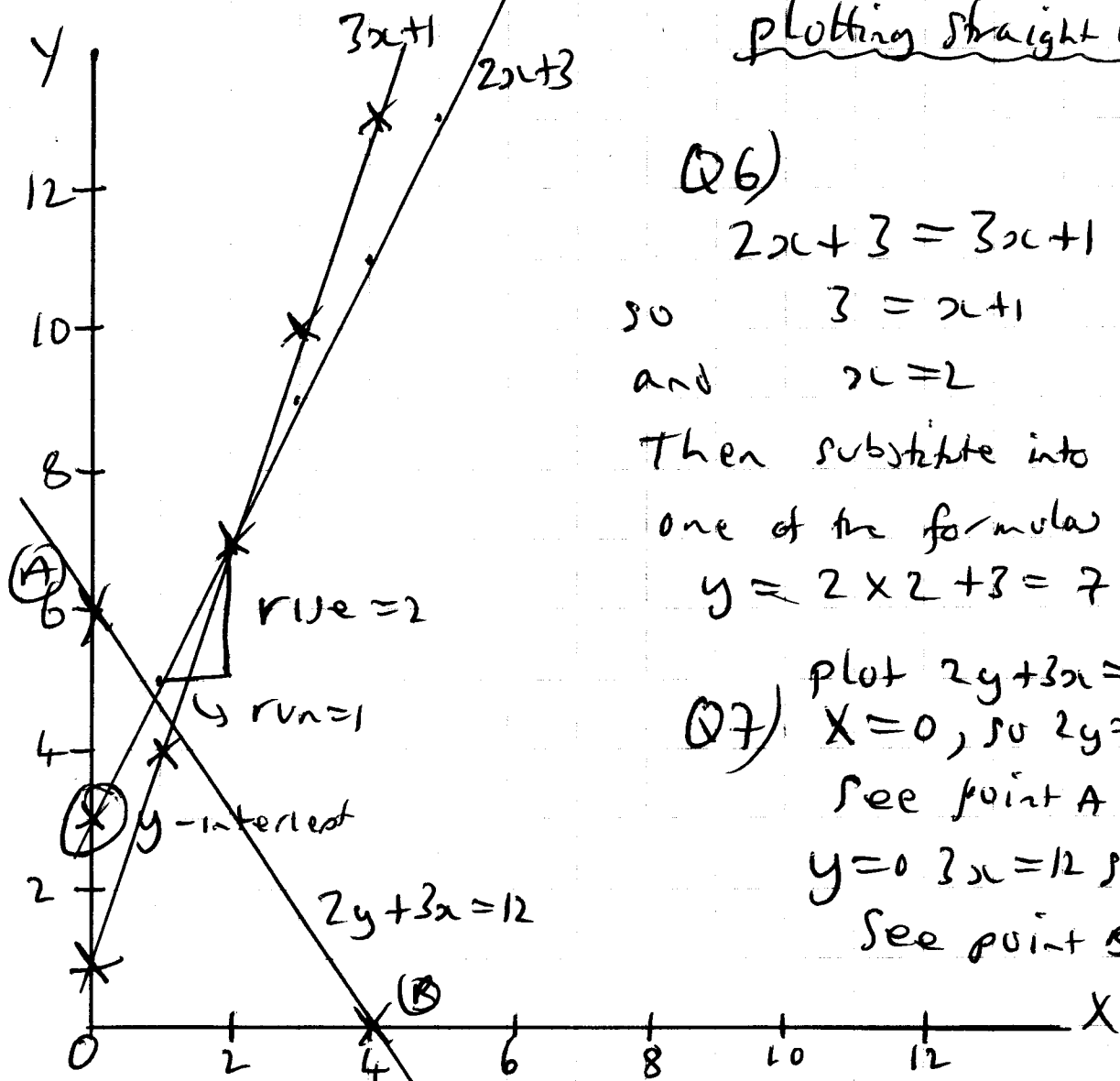
3) (5, 7) (or (2, 9))

6) Area is ~~6~~ $3 \times 2 = \frac{6 \text{ cm}^2}{2}$
 $\approx 3 \text{ cm}^2$

Worksheet 20: solutions

(13)

plotting straight lines



Q6)

$$2x + 3 = 3x + 1$$

so $3 = x + 1$

and $x = 2$

Then substitute into one of the formulas

$$y = 2 \times 2 + 3 = 7$$

Q7) plot $2y + 3x = 12$
 $x = 0$, so $2y = 12$, $y = 6$

See point A

$$y = 0 \quad 3x = 12 \quad \text{so } x = 4$$

See point B.

Q1) gradient = $\frac{\text{Rise}}{\text{Run}} = \frac{2}{1} = 2$

Q2) y-intercept has value +3

Q3) formula is $y = 2x + 3$
 ↙ gradient
 ↖ intercept

Q4) $y = 3x + 1$ has y intercept +1 and you go along 1 and up 3. (see plot with crosses)

Q5) Lines cross at (2, 7)

Worksheet 20: solution

(14)

Plotting a quadratic graph from formula

x	1	-2	-1	0	1	2	3	4
y	1	5	0	-3	-4	-3	0	5

$$x=3 \quad y = x^2 - 2x - 3 \\ = 9 - 6 - 3 = 0$$

$$x=1 \quad y = x^2 - 2x - 3 \\ = 1 - 2 - 3 = -4$$

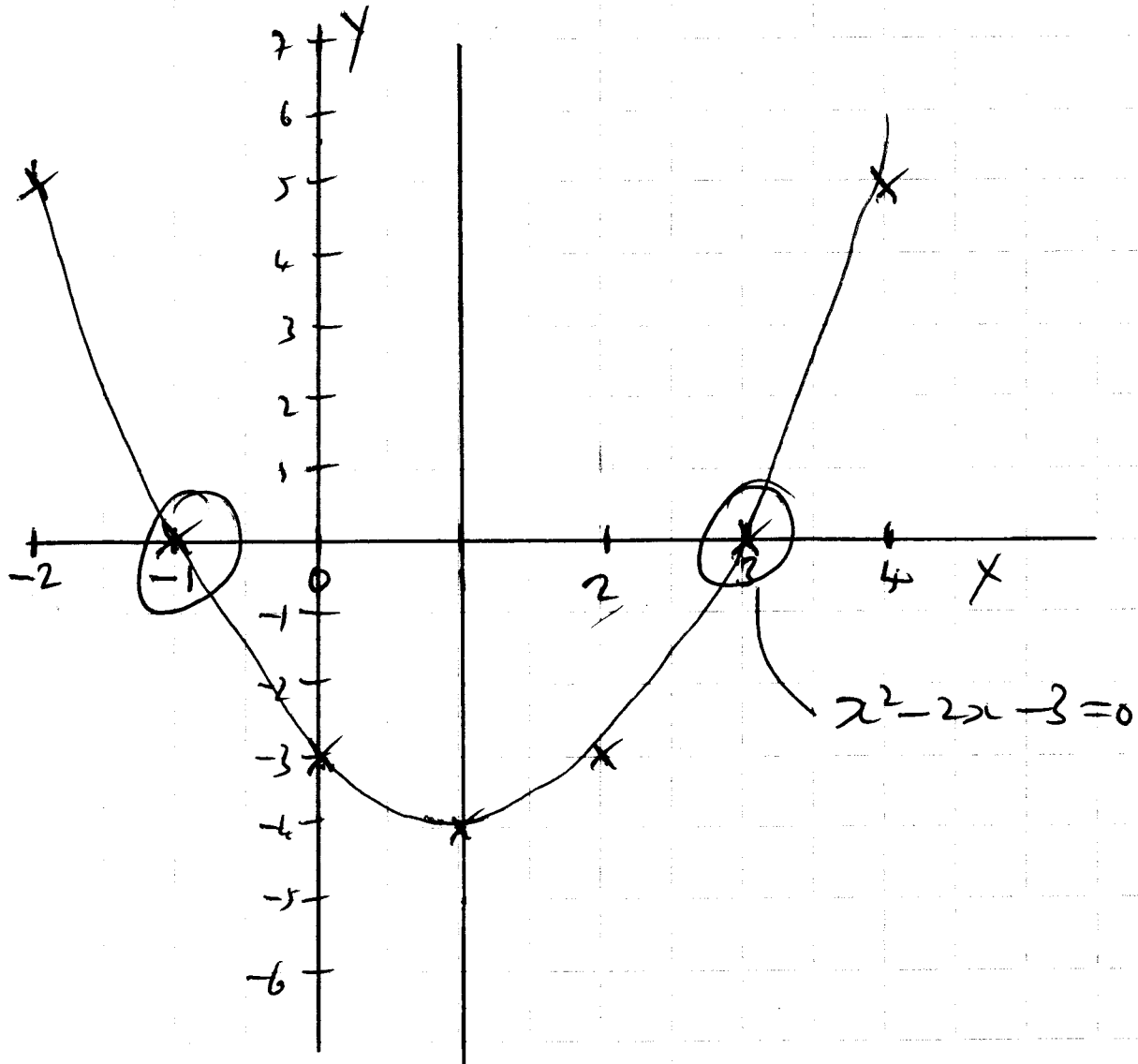
$$x=-2 \quad y = x^2 - 2x - 3 \quad (- - b \bar{a}) \\ = (-2)^2 - 2 \times -2 - 3 \\ = 4 + 4 - 3 \\ = 5$$

See over for plot

$$x=4 \quad y = x^2 - 2x - 3 \\ = 4^2 - 2 \times 4 - 3 \\ = 16 - 8 - 3 \\ = 5$$

Worksheet 20: solution (15)

plot of $y = x^2 - 2x - 3$



c) $x=1$ axis of symmetry

d) challenge $x=3$ and $x=-1$
are points where $y = x^2 - 2x - 3$ crosses
 x axis.